Collective motion II

Leadership and decision making in motion

The relation of collective motion to collective decision making

- If the group is to stay together, individuals constantly have to make decisions regarding
  - When and where to forage, to rest
  - How to defend themselves from predators
  - How to navigate towards a distant targets
  - Etc.
- Cost/benefit ratio (from the viewpoint of the members)
  - Preferred outcome usually differs (information, experience, inner state, etc.)
  - "consensus cost": cost paid by the animal who foregoes its preferred behavior in order to defer to the common decision

First studies – two basic types

Despotic system

- One or a few individual decides
- This can increase the efficiency

Egalitarian / democratic

- Members contribute to the outcome about the same degree
- Smaller average consensus cost

- In nature, both types have been observed
- Sometimes mixed (alternating according to the circumstances)
  - Pairs of pigeons, GPS (2006)
    - Small conflict over the preferred direction: consensus (average)
    - Above a certain threshold: one of them becomes the leader or they split up
  - Similar observations: Wild baboons, GPS (2015)
    - They follow the majority of the “initiators” (those starting off in a certain direction). (And not the dominant individuals)
    - If two groups of initiators (with similar size) heading in different directions:
      - If the angle is less than ~90° → the animals compromise
      - Big angle: they choose one direction over the other (randomly)

Models for leadership

- Extension of the “Couzin model”
- No individual recognition, no signaling mechanism
- Non-informed individuals: are not required to know how many and which individuals has information
- Vice versa: Informed individuals are not required to know anything about the information-level of their mates and that how the quality of their information was compared to that of others.

The model:

- Rule 1: highest priority
  - Individuals attempt to maintain a certain distance among themselves by turning away from those neighbors j which are within a certain distance towards the opposite direction:

$$\vec{d}_i(t+\Delta t) = -\sum_{j \neq i} \frac{\vec{r}_j(t) - \vec{r}_i(t)}{\left|\vec{r}_j(t) - \vec{r}_i(t)\right|}$$

$\vec{d}_i$: desired direction of individual i
$\vec{r}_i$: position of particle i
$\vec{v}_i$: direction of unit i

[Couzin, I.D., Krause, J., Franks, N.R., Levin, S.A., 2005. Effective leadership and decision-making in animal groups on the move. Nature 433, 513–516.]

Models for leadership

The model (cont):

- Rule 2

If there are no mates within the range of repulsion, than the individual will attempt to align with those neighbors j, which are within the range of alignment:

→ The desired direction:

$$\vec{d}_i(t+\Delta t) = -\sum_{j\neq i} \frac{\vec{r}_j(t)-\vec{r}_i(t)}{\left|\vec{r}_j(t)-\vec{r}_i(t)\right|} + \sum_{j\neq i} \frac{\vec{v}_j(t)}{\left|\vec{v}_j(t)\right|}$$

$\vec{d}_i$: desired direction of individual i
$\vec{r}_i$: position of particle i
$\vec{v}_i$: direction of unit i

- Corresponding unit vector: $\hat{d}_i(t) = \vec{d}_i(t)/\left|\vec{d}_i(t)\right|$
- Introducing "influence": a portion of the group (p) is given information/motivation about a preferred direction, described by the (unit) vector $\vec{g}$.
- The rest of the group does not have directional preference.

Informed individuals balance their

- social alignment $\hat{d}_i(t)$ (the unit vector of $\vec{d}_i(t+\Delta t) = -\sum_{j\neq i}\frac{\vec{r}_j(t)-\vec{r}_i(t)}{|\vec{r}_j(t)-\vec{r}_i(t)|} + \sum_{j\neq i}\frac{\vec{v}_j(t)}{|\vec{v}_j(t)|}$) and
- preferred direction $\vec{g}_i$

with the weighting factor ω:

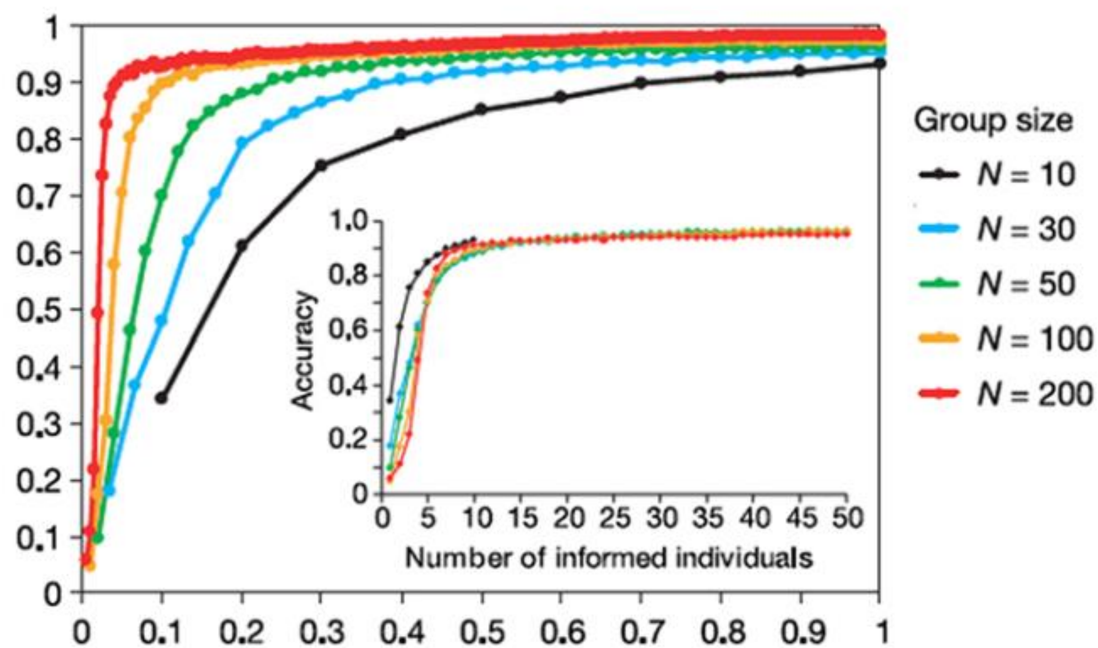
$$\vec{d}_i(t+\Delta t) = \frac{\hat{d}_i(t+\Delta t) + \omega\vec{g}_i}{\left|\hat{d}_i(t+\Delta t) + \omega\vec{g}_i\right|}$$

- ω can exceed 1: the individual is influenced more by its own preferences than by its mates
- "Accuracy" of the group: normalized angular deviation of the group direction around the preferred direction $\vec{g}_i$

Results:

- for fixed group size, the accuracy increases asymptotically as the portion p of the informed members increases

  (…that is…)
- the larger the group, the smaller the portion of informed members is needed, in order to guide the group towards a preferred direction



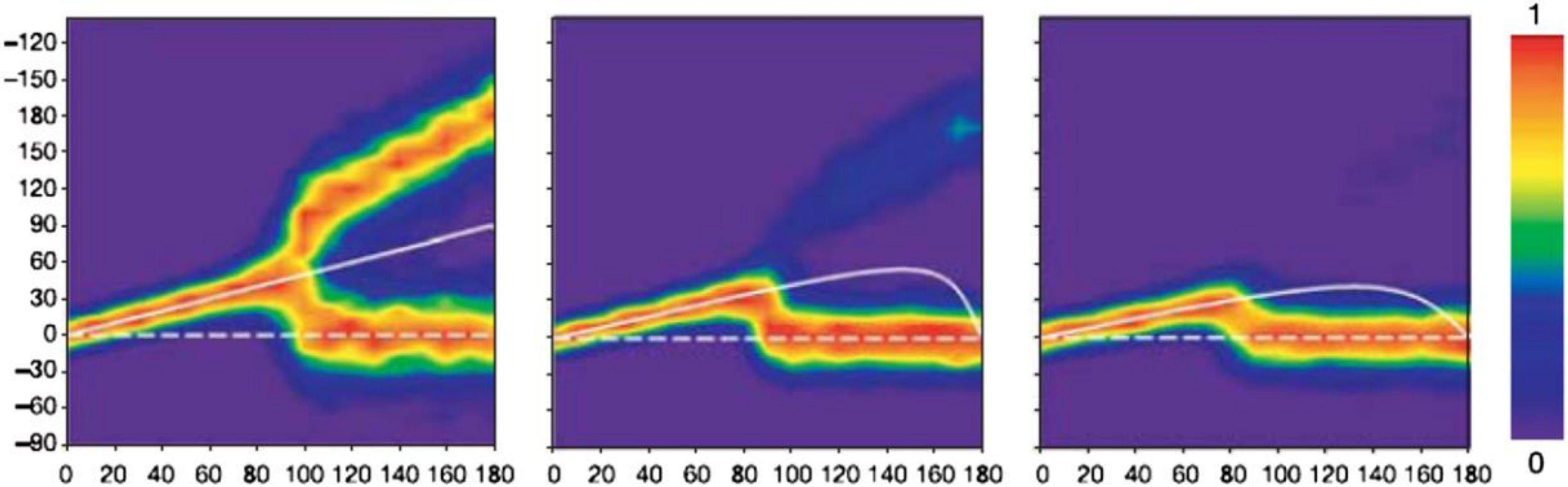
Conflicting preferences

Informed individuals might differ in their preferred direction

1. If the number of individuals preferring one or another direction is equal: the group direction depends on the degree to which the preferred directions differ
   - If it is small: the group will go in the average preferred direction of all informed individuals
   - If it is big: individuals select randomly one or another preferred direction

2. If the number of informed individuals preferring a given direction increases
   - the entire group will go into the direction preferred by the majority (even if that majority is small)

Collective group direction when two groups of informed individuals differ in their preferences - model results



$n_1 = n_2 = 5$

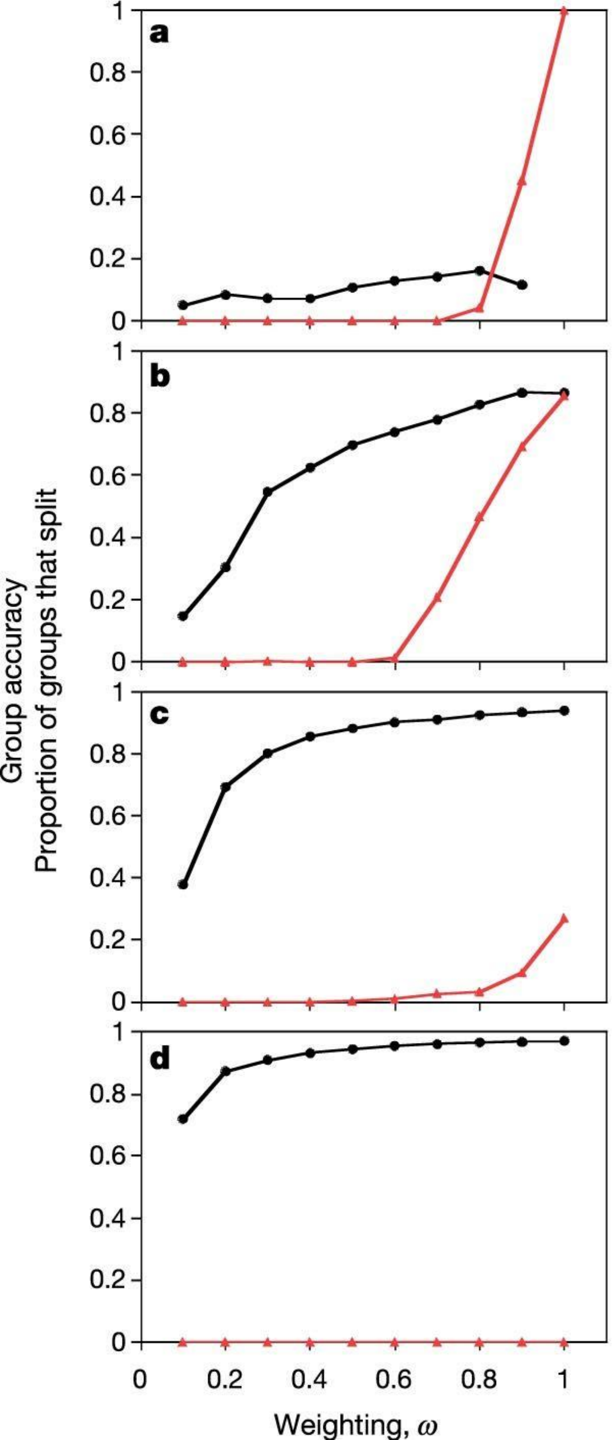
$n_1 = 6$
$n_2 = 5$

$n_1 = 6$
$n_2 = 4$

- Vertical axis: the degree of the most probable group motion.
- The first group (consisting of n_1 informed individuals) prefers the direction characterized by 0 degrees (dashed line),
- The second group (consisting of n_2 informed individuals) prefers a direction between 0 and 180 degrees (horizontal axis)

- Solid white lines are for reference only, representing the direction of the average vector of all informed individuals
- The group consists of 100 individuals altogether

Source: Couzin, I.D., Krause, J., Franks, N.R., Levin, S.A., 2005. Effective leadership and decision-making in animal groups on the move. Nature 433, 513–516.



The influence of the weighting ω of preferred direction

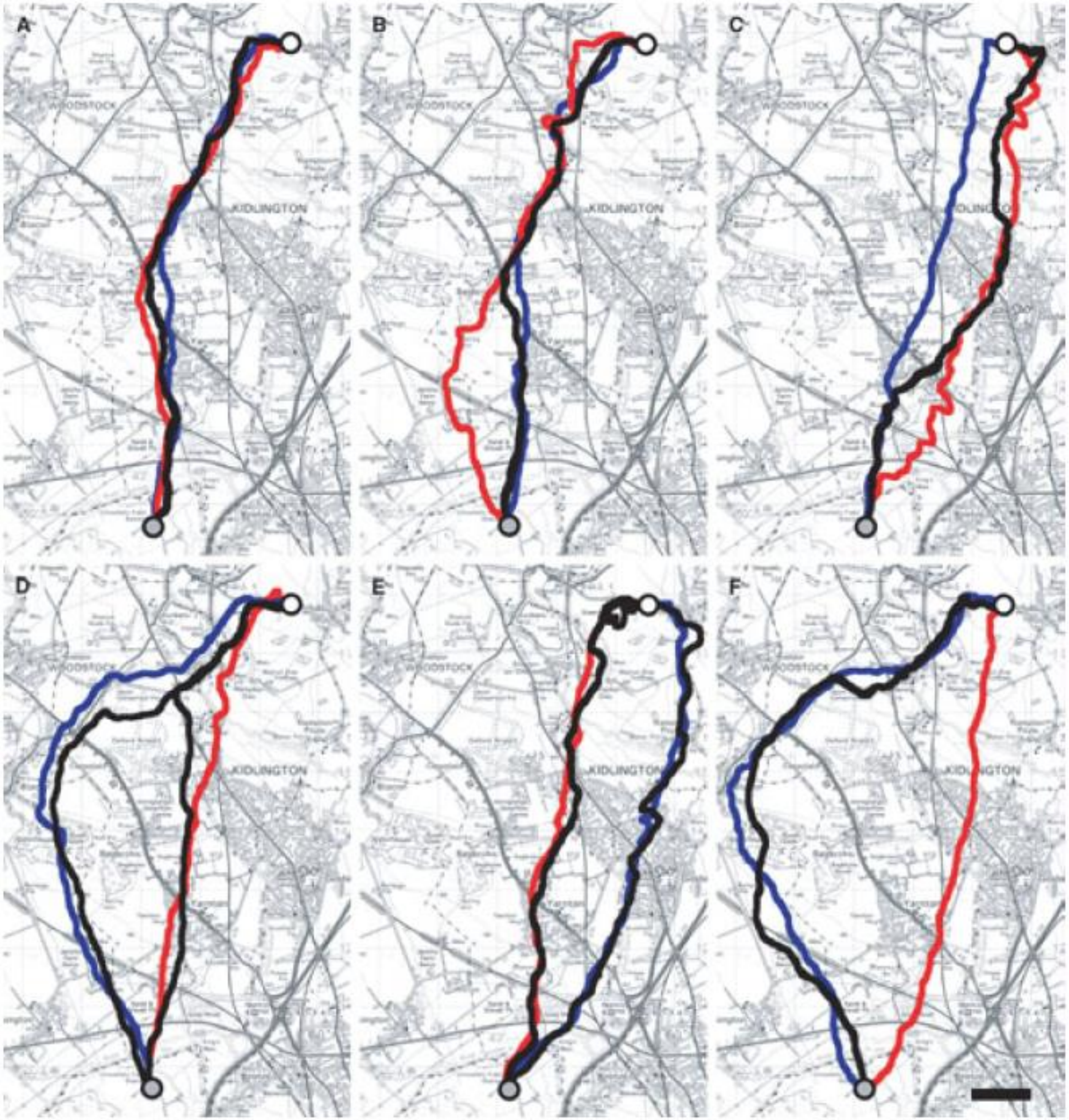
- Informed individuals balance their social alignment $\hat{d}_i(t)$ andpreferred direction $\vec{g}_i$ with the weighting factor ω:

$$\vec{d}_i(t+\Delta t) = \frac{\hat{d}_i(t+\Delta t) + \omega \vec{g}_i}{\left|\hat{d}_i(t+\Delta t) + \omega \vec{g}_i\right|}$$

- ω can exceed 1: the individual is influenced more by its own preferences than by its mates

- Black circles: The accuracy of the group motion
- Red triangles: probability of group fragmentation
- N=50 individuals, *p*: proportion of the informed individuals
  - (a): $p = 0.02$ (1 individual)
  - (b): $p = 0.1$ (5 individuals)
  - (c): $p = 0.2$ (10 individuals)
  - (d): $p = 0.5$ (25 individuals)

Co-released birds and previous recapitulated routes



- Black lines show the flight paths of birds released together.
- Blue and red lines show the previous, stably recapitulated routes of the two individuals comprising the pair.
- (A) Birds remained in a pair throughout the flight, sometimes taking the average route.
- (B) Birds remain in a pair, initially taking an average route, then taking one of the previously established routes.
- (C) Birds remain in a pair and switch between routes.
- (D) Birds initially take a shared, average route, then split and return to their previous routes.
- (E) Birds split at release and fly along their previous routes.
- (F) Birds fly along one of the two previous routes

Further elaboration of the model: introducing the "social importance factor"

- h: strength of the effect of a given individual on the group movement
- higher h implies bigger influence
- varies with each agent

$$\vec{d}_i(t+\Delta t) = -\sum_{j\neq i} h_j \frac{\vec{r}_j(t)-\vec{r}_i(t)}{\left|\vec{r}_j(t)-\vec{r}_i(t)\right|} + \sum_{j\neq i} h_j \frac{\vec{v}_j(t)}{\left|\vec{v}_j(t)\right|}$$

Freeman, R., Biro, D., 2009. Modelling group navigation: dominance and democracy in homing pigeons. The Journal of Navigation 62, 33–40.

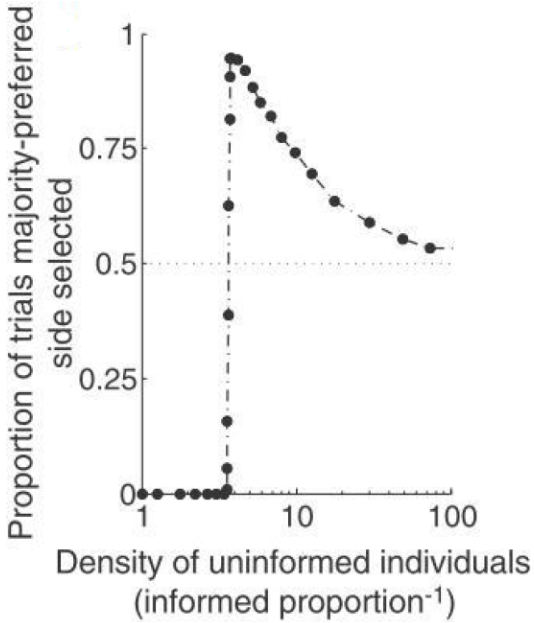
The role of uninformed individuals – simulations vs. experiments

- **Question**: under what conditions can a self-interested and strongly opinionated minority exert its influence on group movement decisions?

- Simulations:
  - Based on the "Couzin model"

$$\vec{d}_i(t+\Delta t) = \frac{\hat{d}_i(t+\Delta t) + \omega \vec{g}_i}{\left|\hat{d}_i(t+\Delta t) + \omega \vec{g}_i\right|}$$

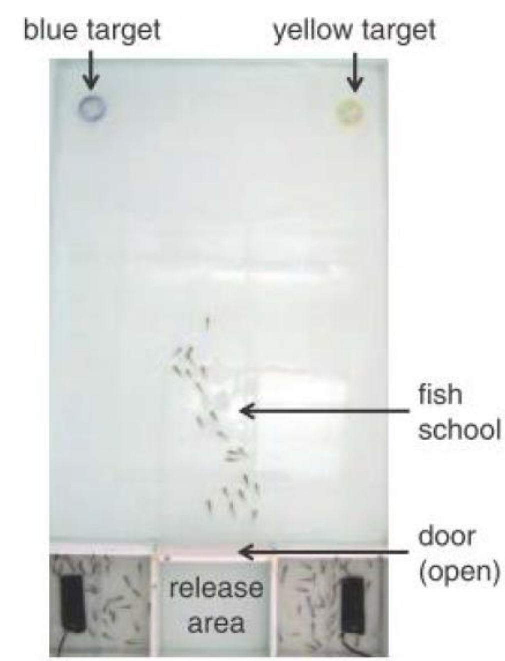
  - If all individuals are biased:
    - If the strength of the majority preference (ω_1) is equal to or stronger than the minority preference (ω_2), the group has a high probability of reaching the majority-preferred target.
    - Increasing ω_2 (beyond ω_1) can result in the minority gaining control
  - If there are uninformed individuals ($\omega_3 \approx 0$):
    - (most animal groups are like this)
    - Adding uninformed individuals tends to return control spontaneously to the numerical majority
    - this effect reaches a maximum and then begins to slowly diminish, and eventually, noise will dominate



A sharp transition from a minority- to majority-controlled outcome in the model as the density of uninformed individuals is increased.
($\omega_{minority} > \omega_{majority}$)

Experiment

- golden shiners
- two groups of initiators (with sizes N_1 and N_2) with different preferred directions (blue and yellow target)
- some did not have direction preference
- $N_1 > N_2 \quad (N_1 = 6$ and $N_2 = 5)$
- Among the trained fish, ω_{yellow} is “by nature” $> \omega_{blue}$
- Simulations predict a large effect for a relatively small number of naïve individuals; $N_3 = 0, 5, 10$.
- When all individuals exhibit a preference ($N_3 = 0$) then the minority N_2 dictates the consensus (even though the fish trained to the blue target are more numerous).
- When untrained individuals are present, they increasingly return control to the numerical majority N_1.
- If individuals with the stronger preference were also in the numerical majority: the majority was more likely to win (72% of trials overall), and the presence of uninformed individuals had no effect



Experimental set-up

Couzin et al, 2011, Uninformed individuals promote democratic consensus in animal groups. Science, 334(6062):1578-80

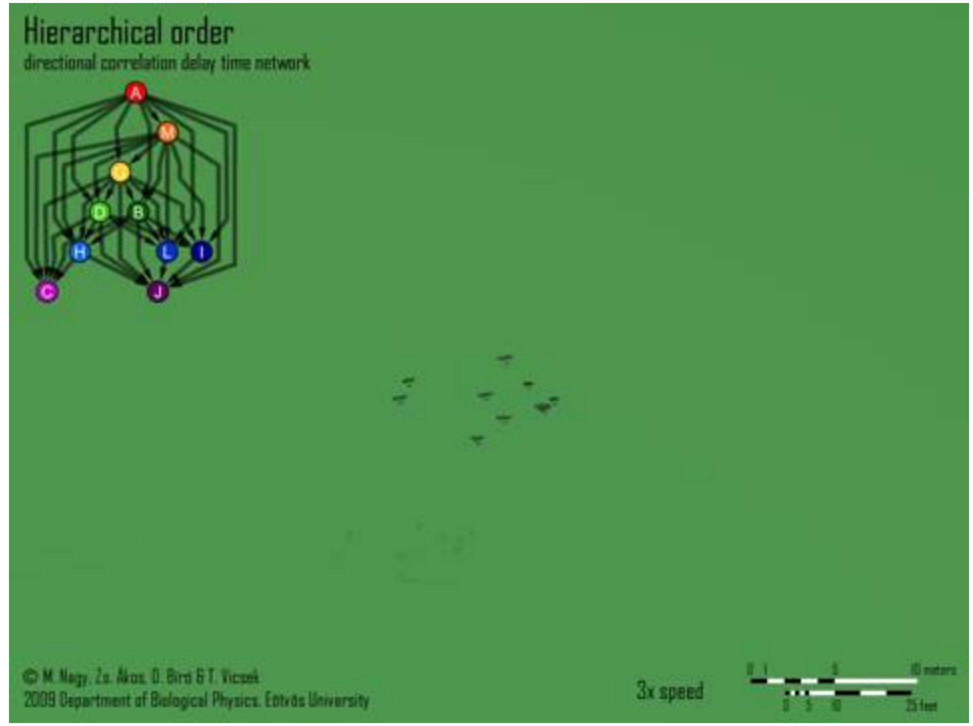
Lessons

- Leadership might emerge from the differences of the level of information possessed by the group members
- information can be pertinent → leadership can be transient and transferable too

Experiments with homing pigeons

- 10 homing pigeons flying in flocks
- high-precision lightweight GPS
- Two kind of flights were recorded:

1. spontaneous flights near the home loft ("free flights") and
2. during homing following displacement to distances of approximately 15 km from the loft ("homing flights")

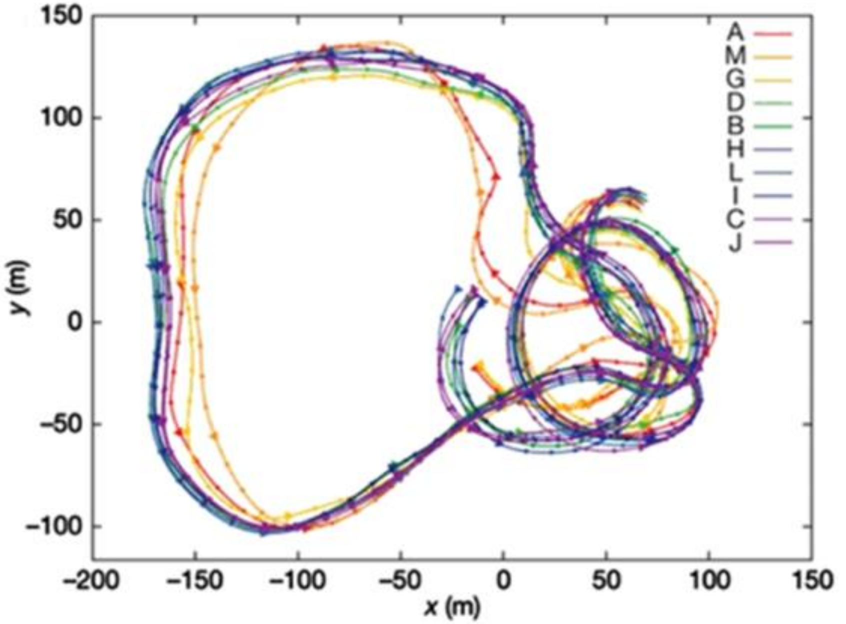


Trajectories of a flock of nine pigeons during a homing flight

Nagy M, Ákos Zs, Bíró D, Vicsek T: *Hierarchical group dynamics in pigeon flocks*, Nature **464**, 890–893, 2010

Analysis

- Goal: to find out how homing pigeons navigate collectively (leadership hierarchy)
  - The *influence* of the birds' behavior on its fellow flock members and on the flock
- → temporal relationship between the bird's flight direction and those of others
- "Leading event": when a bird's direction of motion was "copied" by another bird, delayed in time.



2-minute segment from a free flight performed by a flock of ten pigeons in the vicinity of the loft. The smaller and the larger dots indicate every 1s and 5s, respectively. Each path begins near the center of the plot. Letters refer to bird identity.

This was quantified by determining the directional correlation delay time ($\tau^*{}_{ij}$) (measured in seconds) from the maximum value of the directional correlation function

$$C_{ij}(\tau) = \langle \vec{v_i}(t) \cdot \vec{v_j}(t+\tau) \rangle$$

brackets: time average for each pair of birds i, j

Yielding the directional correlation function

a

- light grey: bird i
- dark grey: bird j
- For each pair ($i \neq j$) the directional correlation function is

$$C_{ij}(\tau) = \langle \vec{v_i}(t) \cdot \vec{v_j}(t+\tau) \rangle$$

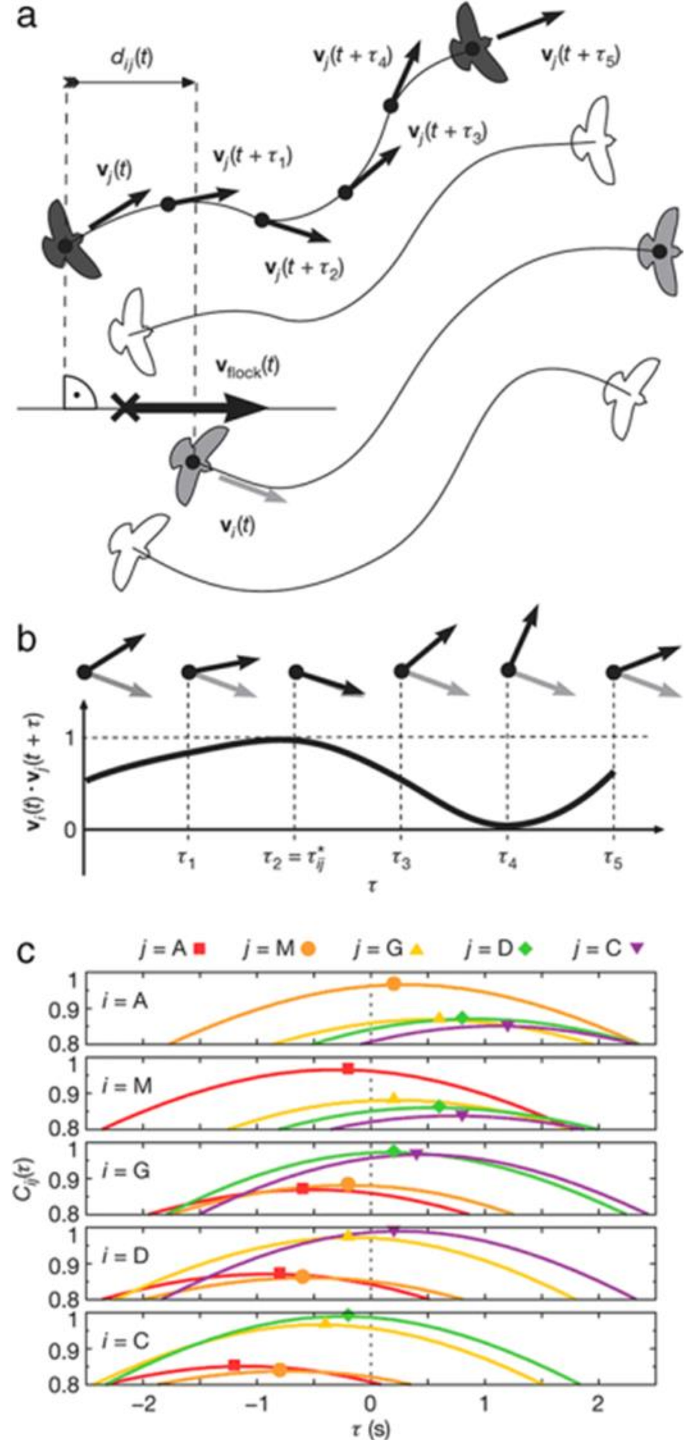
- The arrows show the direction of motion, $\vec{v_i}(t)$

b

- Visualization of scalar product of the normalized velocity of bird i at time t and that of bird j at time $t + \tau$. In this example bird j is following bird i with correlation time ${\tau_{ij}}^*$.

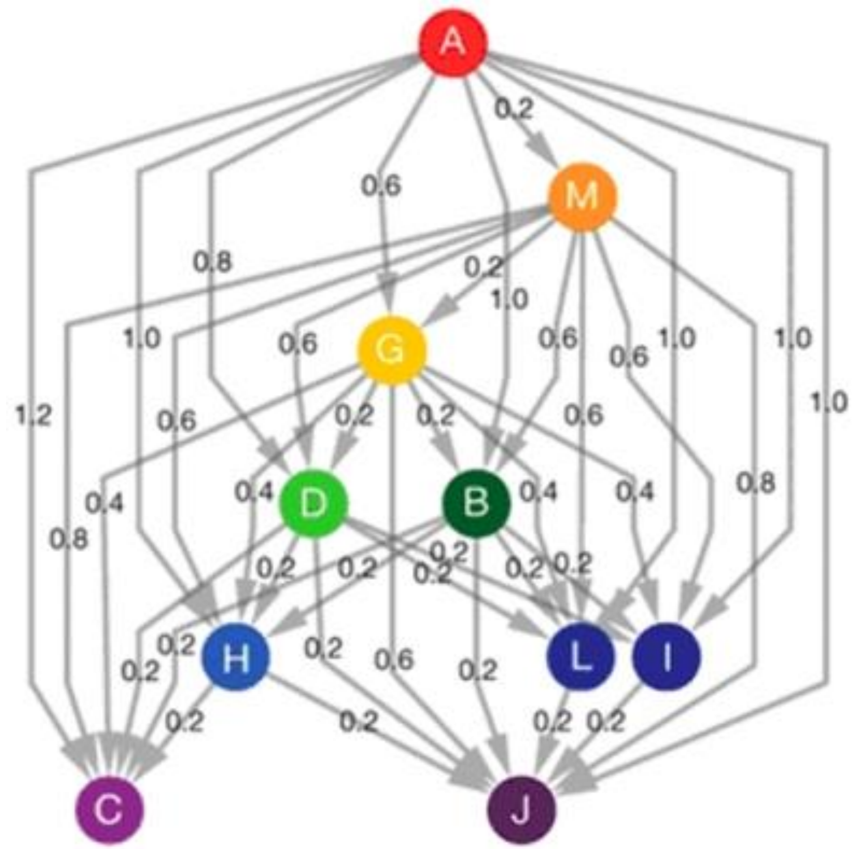
c

- The directional correlation function $C_{ij}(\tau)$ during the flock flight.. For more transparency only the data of birds A, M, G, D and C (in the order of hierarchy for that flight) are shown. The solid symbols indicate the maximum value of the correlation function, ${\tau_{ij}}^*$.
- These ${\tau_{ij}}^*$ values were used to compose the directional leader-follower networks.



Hierarchical leadership network generated for a single flock flight

- The directed edge points from the leader to the follower (i.e., the average directional correlation delay time for that pair, $\overline{\tau_{ij}}$, is positive);
- Values on edges show the time delay (in seconds) in the two birds' motion.
- For pairs of birds not connected by edges directionality could not be resolved at $C_{min} = 0.5$.



Leadership vs. dominance

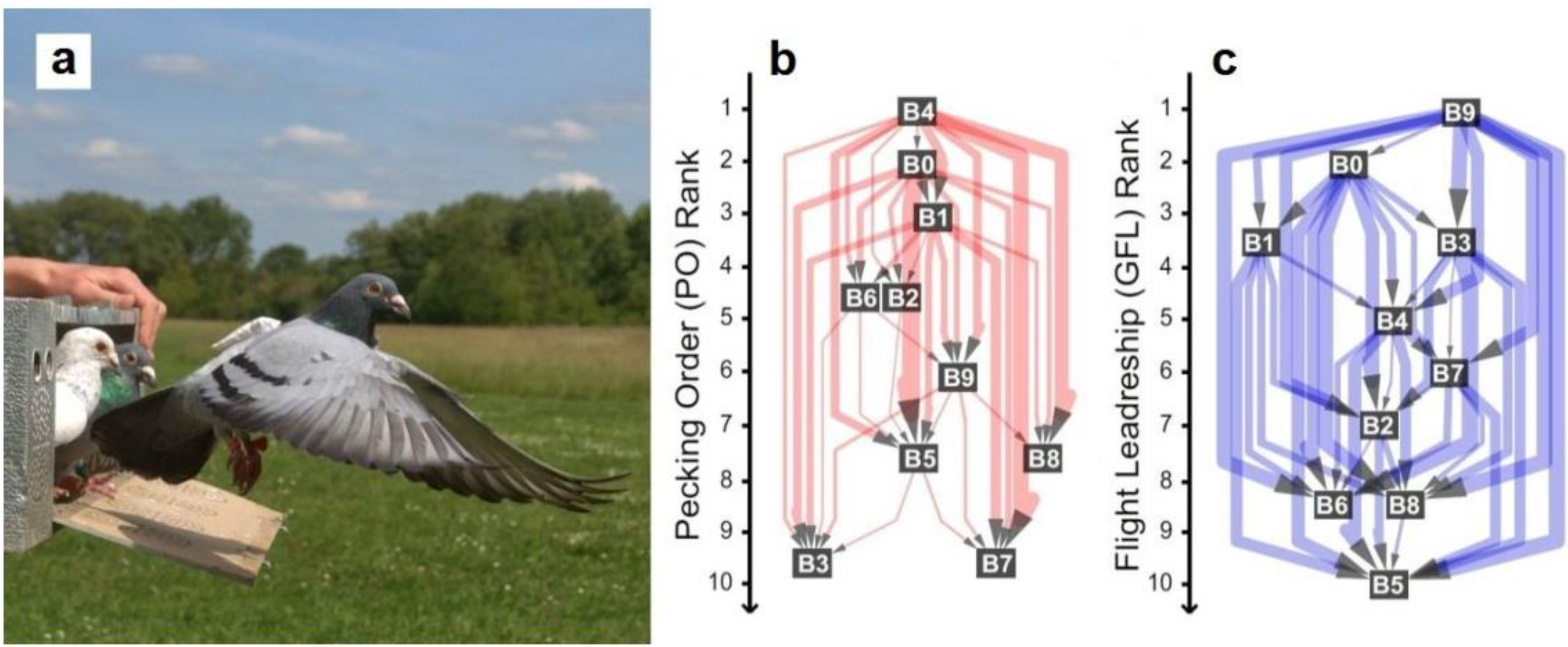
- Assumption: dominant individuals are the leaders.
- Dominance hierarchy
  - Social animals organize themselves into hierarchical groups
  - Regulate access to resources.
  - The mechanism is simple: higher ranked individuals have primacy compared to their lower level mates.
  - As one advances in the evolutionary tree, the structure of the dominance hierarchy gets more and more pronounced and complex, accompanied by more and more sophisticated strategies by which individuals try to get higher and higher ranks.
  - Chimpanzees:
    - coalition formation
    - manipulation
    - exchange of social favors
    - adaptation of rational strategies

Systematic study on dominance hierarchy vs. LFH

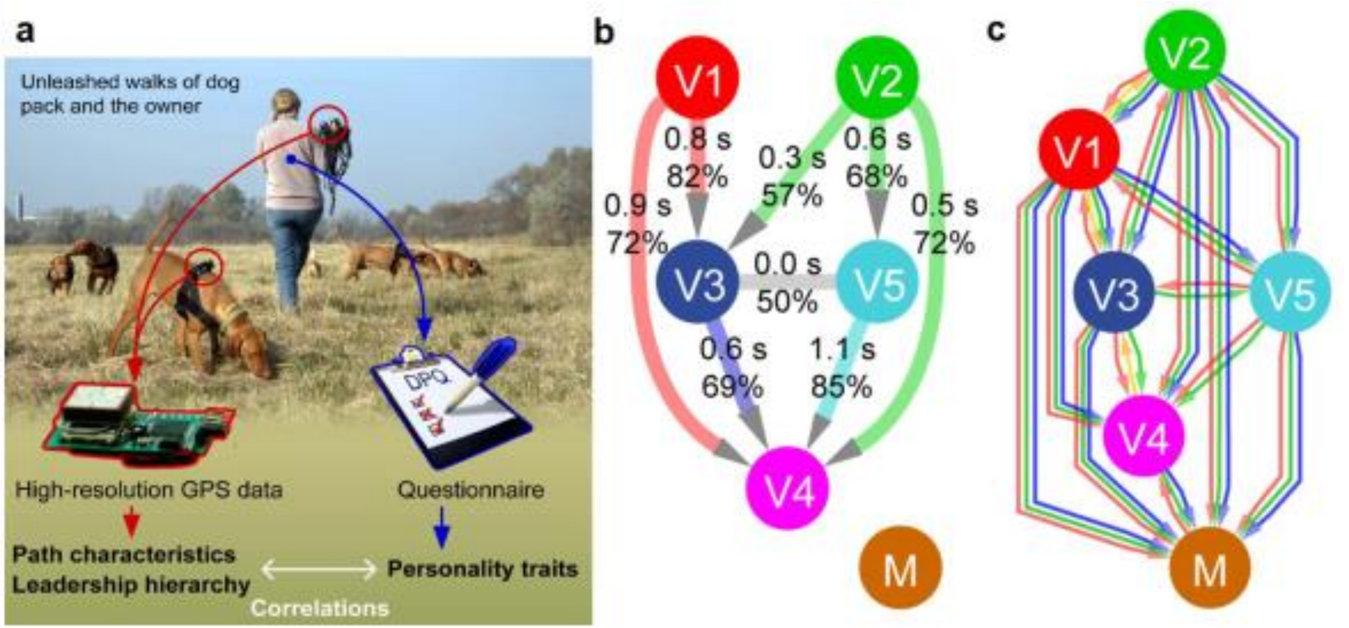
- Flock of 10 pigeons
- L-F hierarchy was determined based on the directional correlation function analysis
- Dominance hierarchy was also determined (in the same group), based on computer-vision methods
- The first automated analysis of dominance relationships
- Both structure is clearly hierarchical

Leadership vs. dominance – Results

- dominance and leadership hierarchies are completely independent of each other
- They can coexist within the same group without any kind of conflict: when it comes to collective travel those will lead the group who have better navigation skills (or information, etc.) and when it comes to feeding, mating, etc., dominance will decide.
- Hierarchy is context-dependent!



Dominance vs. leadership hierarchy in dogs



- 6 dogs, belonging to the same household
- GPS logs during more than a dozen 30- to 40-minute unleashed walks, accompanied by their owner
- All the dogs were “Vizsla”, except for the one marked with “M”, which was a mixed-breed. This dog did not participate in the vizsla-network.

b) Leader-follower hierarchy

- The basis of creating the L-F NW was the directional delay time analysis
- The directed links: point from the leader towards the follower.
- Characteristic delay times are shown on the arrows (upper values).
- Lower values indicate the portion that the leader of that pair was actually leading.

c) Dominance network of the dogs

- derived from a questionnaire.
- The arrows point from the dominant individual towards the subordinate.
- The colors represent the context of the dominance:
  - red: barking,
  - orange: licking the mouth,
  - green: eating
  - blue: fighting.

“How much” knowledge is enough?

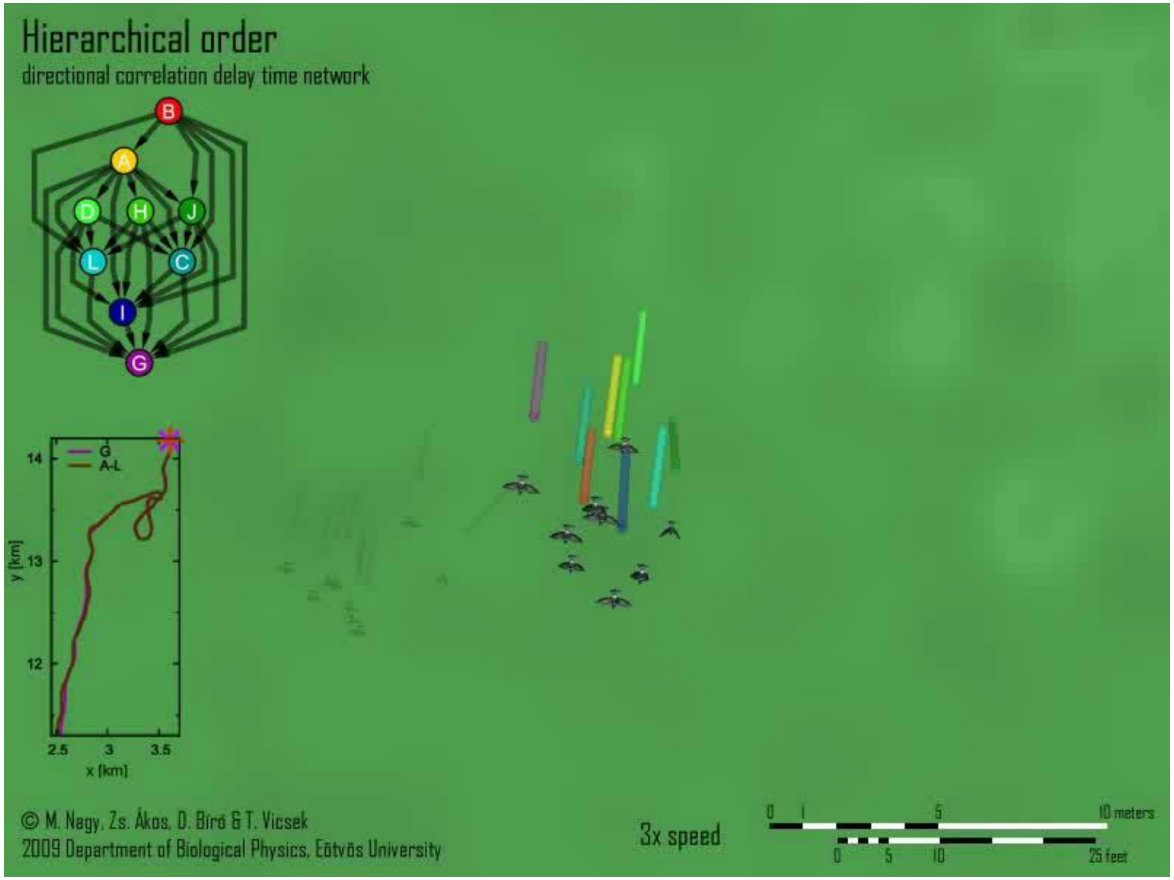
•*high resolution GPS data*

•*hierarchy* of their *leading-following behavior*

• Why do an individual follow an other?

• The ones that are being followed are simply more self-willed or they are better informed?

•How accurate knowledge is needed to reach the target? Etc.



Hierarchical group dynamics in pigeon flocks, M. Nagy et al. *Nature* **464**, 890-893 2010

Formulating the problem:

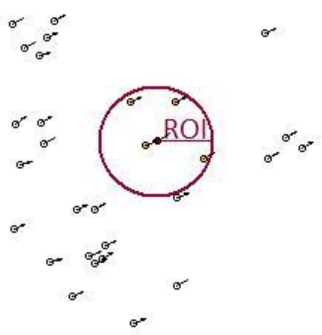
- Given a flock of boids and a pre-defined target
- The flock has to reach the target (together) in the shortest possible way
- The units interact with each other
- The average knowledge is restricted

Question: how to distribute the available amount of knowledge among the group members in order to achieve the best group-performance?

New direction depends on:

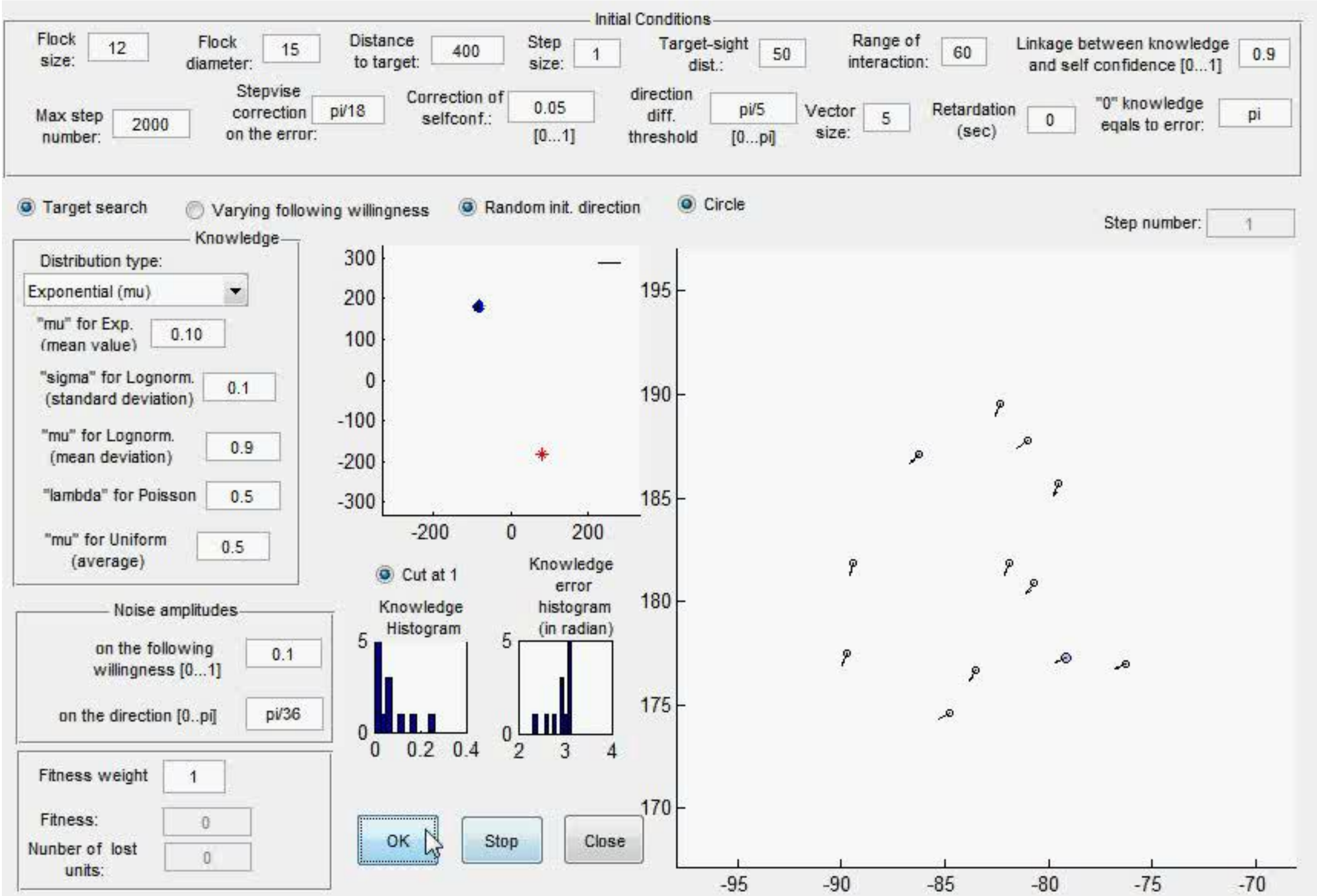
1. The average direction of neighbors (units within the “Range of Interaction, ROI”) $\langle \vartheta_j^t \rangle_R$
2. Own estimation $\theta_i^t + \eta_i^t$
3. Noise ξ_i^t

(Discrete time, constant speed magnitude)

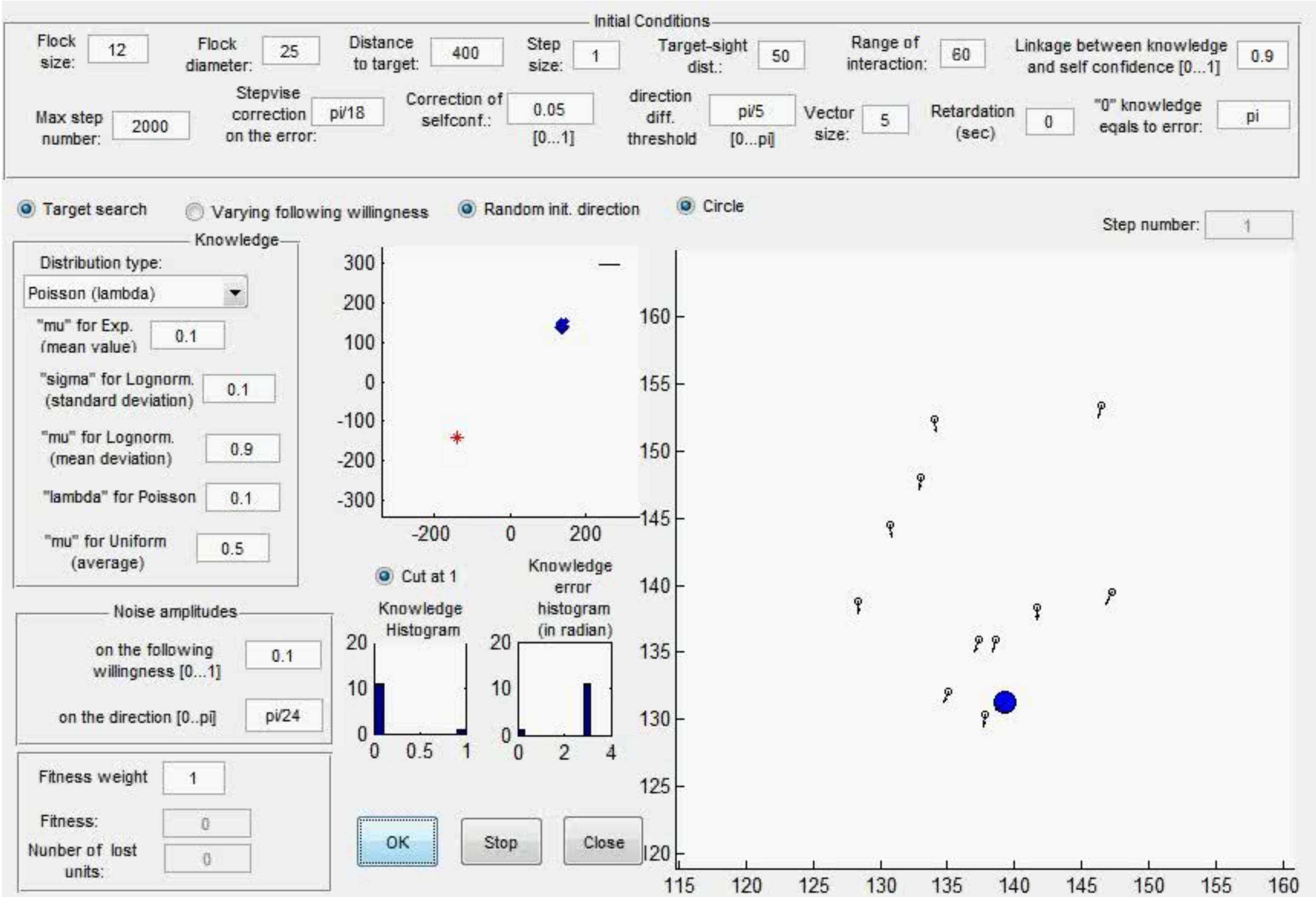


$$\vartheta_i^{t+1} = (1-\lambda_i)(\theta_i^{t+1} + \eta_i^{t+1}) \oplus \lambda_i \left\langle \vartheta_j^t \right\rangle_R \oplus \xi_i^{t+1}$$

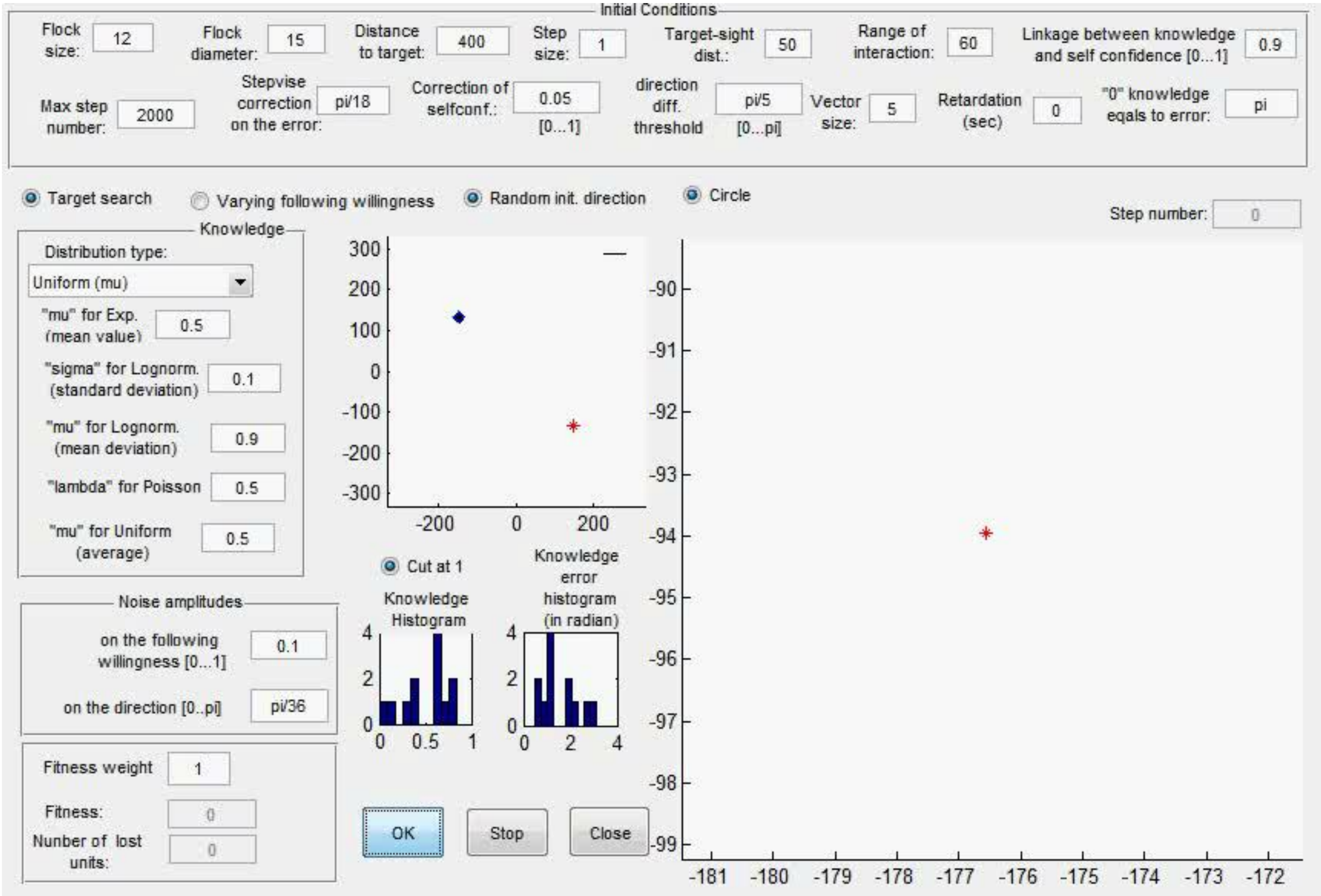
λ_i : a parameter expressing how disposed boid i is to follow others. “Pliancy”
ϑ_i^t : the direction of boid i at time-step t
θ_i^t : the proper direction from boid i towards the target at time-step t
η_i^t : the actual estimation error of boid i at time-step t
ξ_i^t : random noise. $|\xi_i^t| \leq \Xi$ where Ξ is the noise amplitude.
$\oplus$: direction–summation
$\left\langle \vartheta_j^t \right\rangle_R$: the average direction of the boids j being within the range of interaction, R of boid i at time-step t



Flock size = 12, Exponential knowledge distribution, μ=0.1, coded in MatLab.



Flock size = 12, “Two-valued” knowledge distribution, μ=0.1, coded in MatLab.



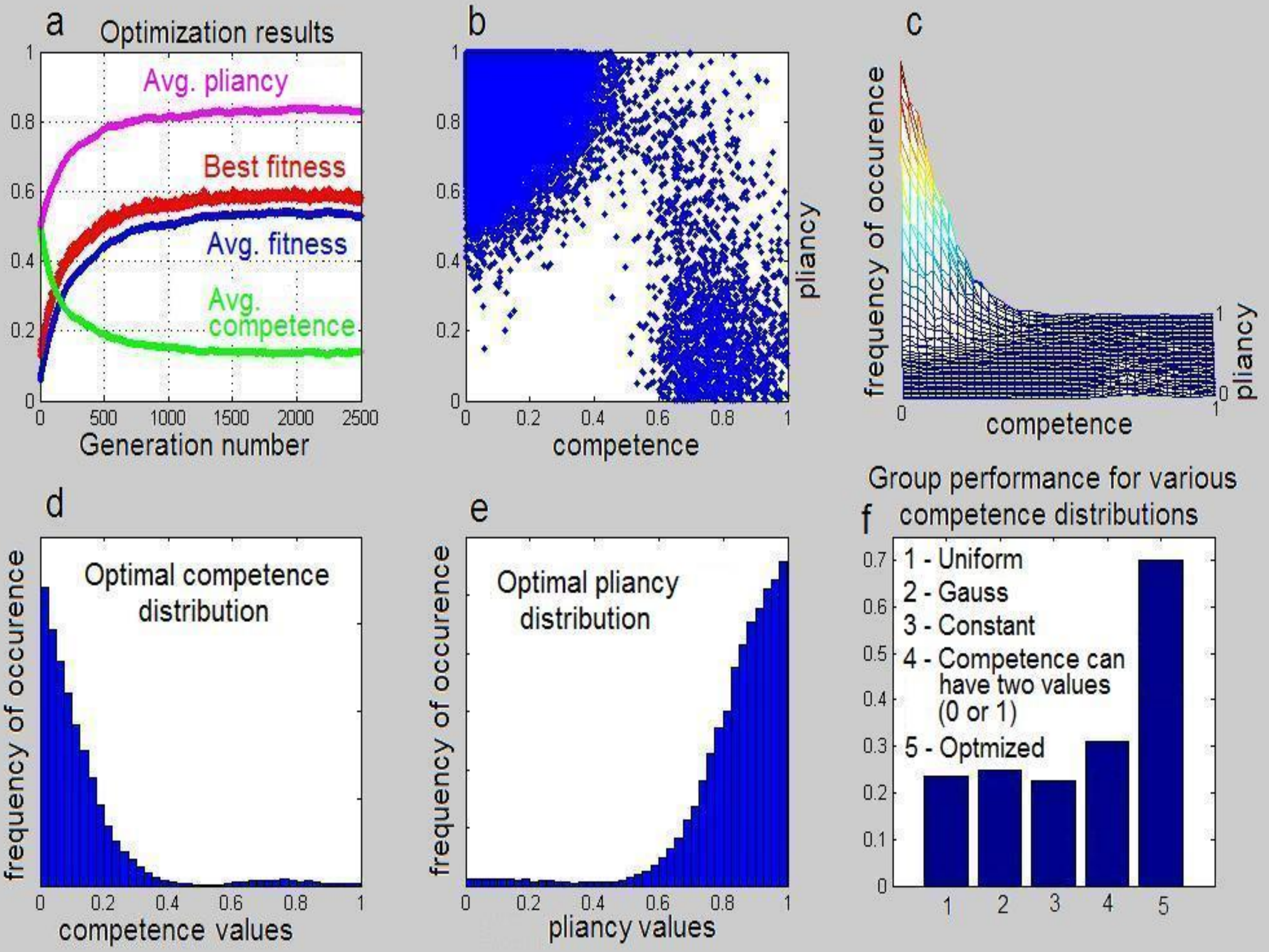
Flock size = 12, Uniform knowledge distribution, μ=0.5, coded in MatLab.

Conclusions of the simulations:

- The *average* knowledge level can be surprisingly small
  - the individual estimations are very imprecise,
  - the knowledge value of most boids can be zero or near-to zero

- The way knowledge is distributed has a huge effect

- It helps, if
  - the units pay attention for their neighbors' movement
  - the pliancy and the knowledge values are inversely related



a
Optimization results
Avg. pliancy
Best fitness
Avg. fitness
Avg. competence
Generation number
b
competence
pliancy
c
frequency of occurence
competence
pliancy
d
Optimal competence distribution
frequency of occurence
competence values
e
Optimal pliancy distribution
frequency of occurence
pliancy values
f
Group performance for various competence distributions
1 - Uniform
2 - Gauss
3 - Constant
4 - Competence can have two values (0 or 1)
5 - Optmized

Case study: Pedestrian motion; Models and their relations

- Always 2D (↔vehicle 1D)

- Traffic models are usually categorized according to the scale of the variables of the model:
  - Macroscopic,
  - Microscopic, and
  - Mesoscopic

Fredrik Johansson, Microscopic Modeling and Simulation of Pedestrian Traffic, Department of Science and Technology, Linköping University, 2013

Macroscopic models / continuum dynamic approach

- Describes the macroscopic, or average, properties of the system
- Assumes that traffic can be regarded as a *fluid*, or continuum, disregarding the fact that it is composed of discrete entities such as cars or pedestrians
  - No explicit reference to the underlying microscopic nature, → no personal preferences
  - Central assumption:
    - no (sufficiently little) significant information is lost when the microscopic details are averaged out
    - the units are identical, unthinking elements
  - successful approach in physics
  - Bit less well founded in traffic modeling, but has been successful, primarily in car traffic modeling
- The basis of fluid dynamic models of pedestrian traffic is the two dimensional continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \boldsymbol{q} = 0$$

where ρ : mean density ($\rho = \rho(\boldsymbol{r}, t)$),

$\boldsymbol{q} = \rho u$: mean flow ($\boldsymbol{q} = \boldsymbol{q}(\boldsymbol{r}, t)$),

u : mean speed (the assumption that *u* is a function of the density, comes from observations)

Mesoscopic models

- Each individual is represented individually and can have individual properties (↔ Macroscopic)
- But the individual walker's behavior is still determined by average quantities

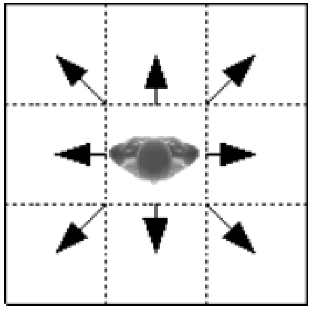
Microscopic models

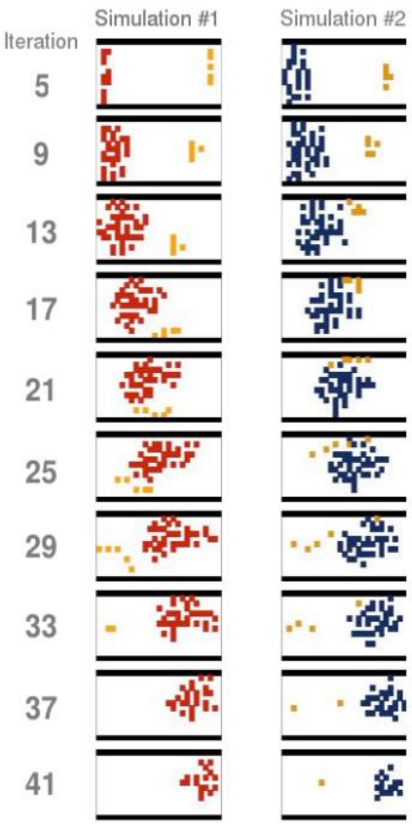
- describe every individual walker and its interaction with other walkers and the environment
- there is no averaging process → the heterogeneity of the population can be explicitly included (personal drives, motivations, preferred directions, etc.)

- Four basic types (partially overlapping, not well defined)
  1. cellular automaton based models
  2. agent based models
  3. game theoretic models
  4. force based models (Social force model)

(1) Cellular automation based models

- Very first models (1980’s), but still in use
- Discrete in space and time
- Each unit is a cell, either occupied by a pedestrian (or obstacle) or empty
- At each time step, pedestrians move into one of the neighboring cells or stay where they are.
- Limitation:
  - the size of a walker is fixed and constant over the population
  - Discrete size of movement at a time
    (but different speeds and goals can be considered)
- Pro-s:
  - Computational efficiency
  - Simple update rules → some general properties are easy to obtain
  - The grids can be refined
- One of the earliest models: Gipps and Marksjö (1985): (the “basics”)
  - grid with quadratic cells
  - The preferred next cell is the one that reduces the remaining distance to the walker’s destination the most
  - The navigation is modified by the presence of other walkers: repulsive potential around each walker





(2) Agent based models

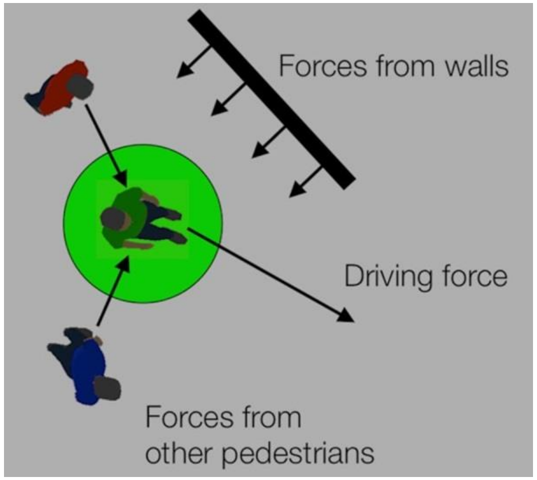
- basically CA models with "very complex" update rules
  - can be either continuous or discrete, both in space and time
  - can be governed by practically any type of behavioral rules.
  - often have a large set of behavioral rules, each dedicated to a specific situation.
  - The update procedure occurs in two steps:
    1. the agent determines the situation it is in by one or several test
    2. Executes the rule connected to that situation
  - Pro: can be very detailed
  - Con: high computational cost, hard to analytically provide properties
  - Can be connected with vision systems

(3) Game theoretic models

- Movement is an “action”
- Each pedestrian plans his/her path according to her beliefs about how other pedestrians will move in the future.
  - Example:
    - Pre defined strategies
    - an empirical distribution over the strategies of other players
    - Etc.

(4) Force based models/social force models (SFM)

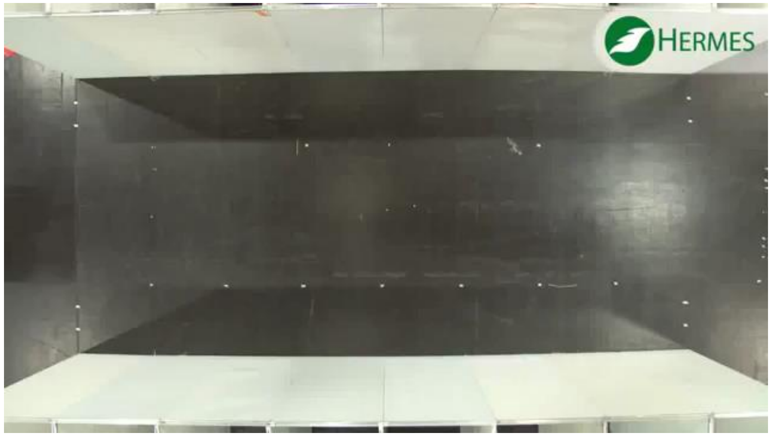
- Helbing and Molnár (1995)
- People walk in crowded environments by using automatic (subconscious) strategies for avoiding collisions and keeping comfortable distances
- These automatic strategies can be encoded as simple behavioral rules

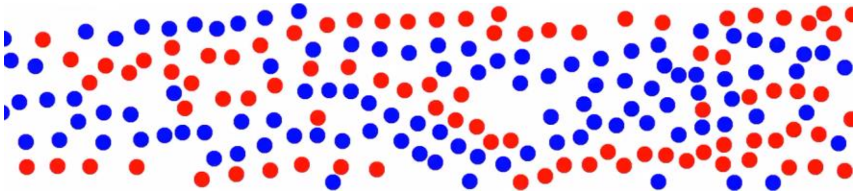


Main idea: the influences of elements of the environment on the behavior of the pedestrians appear as social forces.

- Social forces are not “real” forces (in a Newtonian meaning), rather, are a description of the motivation of the pedestrian to change its velocity, induced by some elements in the environment.
- the effects of several social forces, just like regular forces, are assumed to add as vectors
- Operates in continuous space, allowing detailed representation of the geometry of the environment
- proven to reproduce several well known features of pedestrian traffic:
  - dynamic lane formation in opposing flows
  - oscillations at bottlenecks
  - evacuation scenarios

Dynamic lane formation in opposing flows





Experiment:

Walkers self-organize into lanes to avoid interactions with oncoming pedestrians. This helps them to move faster than is otherwise possible. This happens effortlessly and requires no communication
https://www.youtube.com/watch?v=J4J__lOOV2E

Model:

F. Zanlungo, T. Ikeda and T. Kanda,
Social force model with explicit collision prediction,
Europhysics Letters, Volume 93, 68005

https://www.youtube.com/watch?v=u2kEM2Ed6Xk

An application for SFM: Panic in human crowd

According to the socio-psychological literature the characteristic features of escape panics:

(1) People try to move considerably faster than normal
(2) Individuals start pushing, and interactions become physical.
(3) Moving and passing of a bottleneck becomes uncoordinated.
(4) At exits arching and clogging are observed.
(5) Jams build up
(6) The physical interactions add up and cause dangerous pressures up to 4,450 $^{N}/_{m^2}$ which can bend steel barriers or push down brick walls

Dirk Helbing, Illés Farkas, and Tamás Vicsek: *Simulating dynamical features of escape panic*. Nature 407, 487-490 (2000)

Faster is slower in pedestrian evacuation

Experiment (by GranularLab)

Illustrative video experimentally demonstrating the Faster is Slower effect in pedestrian evacuation through narrow doors. The charts appearing in the vertical direction are spatio-temporal diagrams constructed by taking the lines of pixels displayed by green and stacking them vertically as time evolves. For more information: http://journals.aps.org/pre/abstract/...

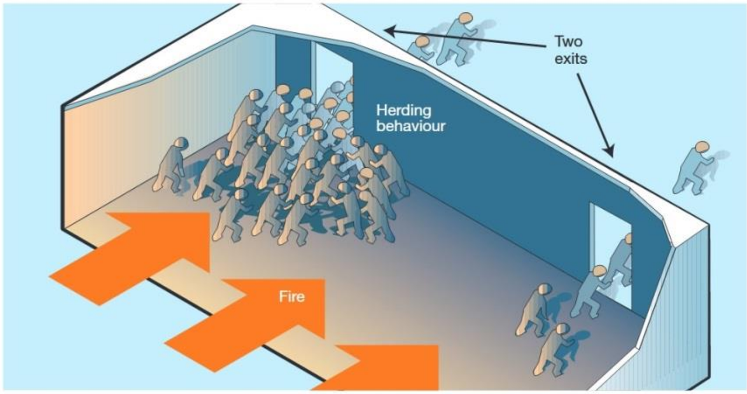
https://www.youtube.com/watch?v=q-k4fCiiMlk

Model: Panic in human crowd

- Many-particle SPP system
- Main assumption: the individual behavior is influenced by a mixture of socio-psychological and physical forces

$$m_i \frac{d\mathbf{v}_i}{dt} = m_i \frac{v_i^0(t)\mathbf{e}_i^0(t) - \mathbf{v}_i(t)}{\tau_i} + \sum_{j(\neq i)} \mathbf{f}_{ij} + \sum_W \mathbf{f}_{iW}$$

N: number of pedestrians (size of the crowd)
m_i: mass of the i-th pedestrian
${v_i}^0$: desired speed of individual i
${\boldsymbol{e}_i}^0$: preferred direction of individual i
$\boldsymbol{v}_i(t)$: actual velocity
τ_i: characteristic („reaction") time of individual i
f_{ij} and f_{iW}: „interaction forces": individual i tries to keep a velocity-dependent distance from other pedestrians j and walls W.



How crowd behaviour affects escape from a smoke-filled room. Previous simulations of pedestrian behaviour in crowds have used a model based on fluid flow through pipes, but these ignored the actions of individuals. According to the individual-centred model of Helbing *et al.*[1], the evacuation of pedestrians from a smoke-filled room with two exits can lead to herding behaviour and clogging at one of the exits. By contrast, a traditional fluid-flow model would predict the efficient use of both exits. A more individual-centred approach is required to reproduce the behaviour of real crowds.

Panic model – cont.

$$m_i \frac{d\mathbf{v}_i}{dt} = m_i \frac{v_i^0(t)\mathbf{e}_i^0(t) - \mathbf{v}_i(t)}{\tau_i} + \sum_{j(\neq i)} \mathbf{f}_{ij} + \sum_W \mathbf{f}_{iW}$$

The psychological tendency of pedestrians i and j to avoid each other: repulsive interaction force:

$$A_i e^{\frac{r_{ij}-d_{ij}}{B_i}} \boldsymbol{n}_{ij}$$

If $d_{ij} < r_{ij}$ then the pedestrians touch each other. In this case two additional forces (after granular interactions):

1. "Body force":
   $k(r_{ij} - d_{ij})\boldsymbol{n}_{ij}$
   counteracting body compression
2. "Sliding friction force"
   $\kappa(r_{ij} - d_{ij})\Delta v_{ij}{}^t \boldsymbol{t}_{ij}$
   impeding relative tangential motion
   $\boldsymbol{t}_{ij}$ is the tangential direction, and
   $\Delta v_{ij}{}^t = (\boldsymbol{v}_j - \boldsymbol{v}_i) \cdot \boldsymbol{t}_{ij}$ is the tangential velocity difference

N: number of pedestrians (size of the crowd)
m_i: mass of the i-th pedestrian
$v_i{}^0$: desired speed of individual i
$\boldsymbol{e}_i{}^0$: preferred direction of individual i
$\boldsymbol{v}_i(t)$: actual velocity
τ_i: characteristic („reaction") time of individual i
f_{ij} and f_{iW}: „interaction forces": individual i tries to keep a velocity-dependent distance from other pedestrians j and walls W.

$\boldsymbol{r}_i(t)$ position of individual i
A_i constant
B_i constant
$d_{ij} = \|\boldsymbol{r}_i - \boldsymbol{r}_j\|$ distance between the pedestrians' center of mass
$\boldsymbol{n}_{ij}$: normalized vector pointing from pedestrian j to i
r_i : the radius of pedestrian i
$r_{ij} = r_i + r_j$ the sum of the radii of pedestrians i and j

κ : constant (large)
k : constant (large)

$g(x)$: zero, if the pedestrians do not touch each other ($d_{ij} > r_{ij}$),
Otherwise equal to the argument x.

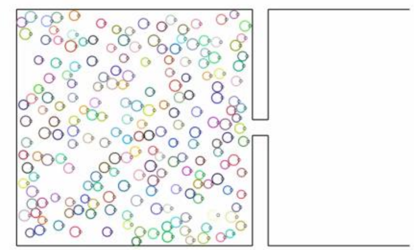
$$f_{ij} = \left\{A_i e^{\frac{r_{ij}-d_{ij}}{B_i}} + k \cdot g(r_{ij} - d_{ij})\right\}\boldsymbol{n}_{ij} + \kappa g(r_{ij} - d_{ij})\Delta v_{ij}{}^t \boldsymbol{t}_{ij}$$

Simulation results with reasonable parameters

1. Transition to incoordination due to clogging.

   The outflow from a room is well coordinated and regular desired velocities are normal.

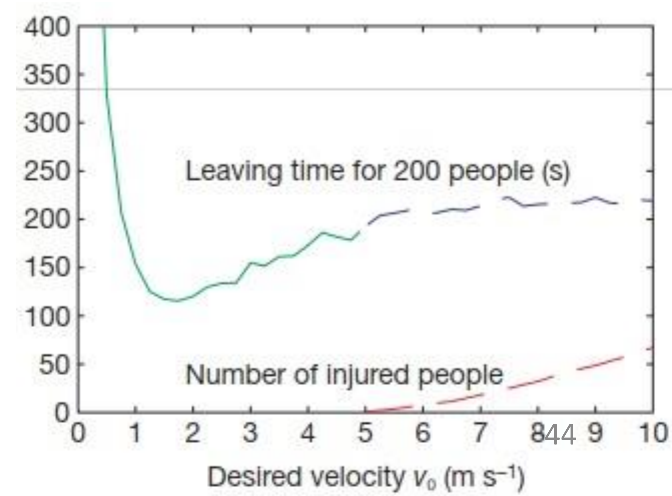
   But for desired velocities above $1.5\ ^{m}/_{s}$ (rush) an irregular succession of arch-like blockings of the exit and avalanche-like bunches of leaving pedestrians when the arches break appear.



Simulation of 200 pedestrians evacuating a 15x15m room passing through a 1meter-wide door at a desired speed of 3.5m/s.
https://www.youtube.com/watch?v=FidqTZiJvRA

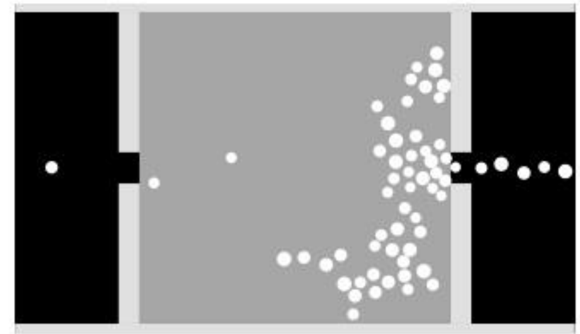
2. “Faster-is-slower” effect due to impatience. Since clogging is connected with delays, trying to move faster can cause a smaller average speed of leaving (κ is large)

   - fire



Simulation results with reasonable parameters

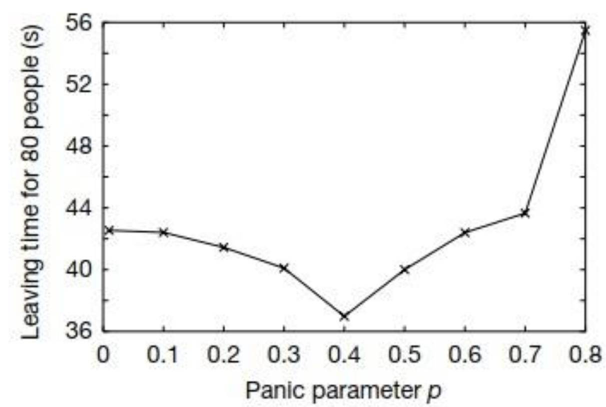
3. **Mass behavior.** Simulated situation: pedestrians are trying to leave a smoky room, but first have to find one of the invisible exits.



Each pedestrian i may either

- select an individual direction $\boldsymbol{e_i}$
- follow the average direction $\langle \boldsymbol{e_j}^0(t) \rangle_i$ of his neighbors j in a certain radius R_i
- mix the two with a weight parameter p_i

$$\mathbf{e}_i^0(t) = \mathrm{Norm}[(1 - p_i)\mathbf{e}_i + p_i \langle \mathbf{e}_j^0(t) \rangle_i]$$



- if p_i is small → individualistic behavior
- if p_i is big → herding behavior
- → p_i is the "panic parameter" of individual i
- Best chances of survival: a certain mixture of individualistic and herding behavior